2) Calculations for Compressed Spring Washers (Reference)

(1) Load and Stress Calculations of Curved Washer

If the Curved Waster is assumed to be a free-supported beam, the following formulas are given:

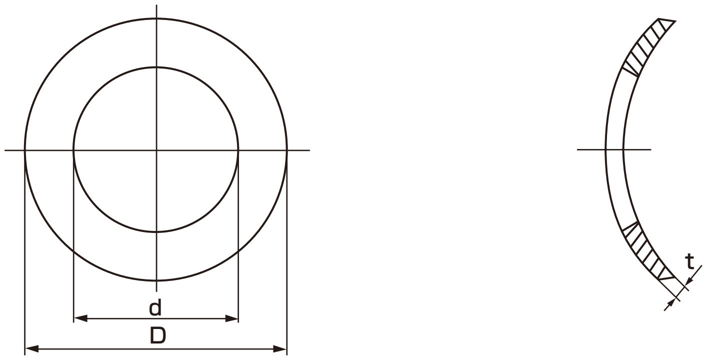


Fig. 1 Curved washer

Load

$$P = \frac{4K_1 E t^3 \delta}{D^2} \quad (1)$$

Stress

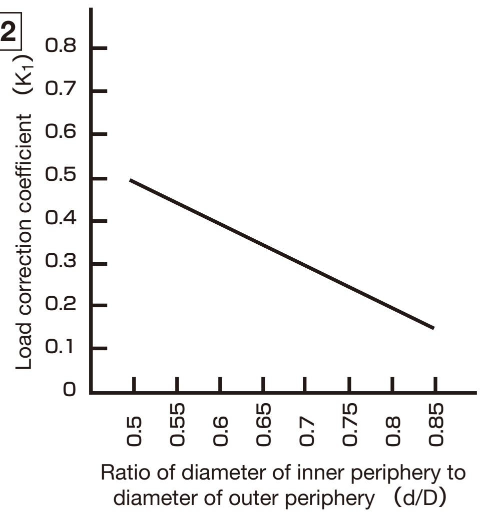
$$S = \frac{1.5P}{K_1 t^2} \quad (2)$$

P : Load (N)
S : Stress (N/mm²)
D : Diameter of outer periphery (mm)
d : Diameter of inner periphery (mm)
t : Plate thickness (mm)
δ : Amount of deflection (mm)
E : Longitudinal elastic modulus (N/mm²) Table 1
K_1 : Load correction coefficient〔=1−d/D〕 Table 2

Table 1 Longitudinal elastic moduli of main materials (E)(N/mm²)

Material	Longitudinal elastic modulus
Carbon spring steel	206000
Stainless steel for spring	181000

Table 2



Notes

There are differences between the calculated and measured values for the formula of deflection and load. Substitution of conditions such as diameters of outer and inner peripheries gives a first-order equation of deflection and load which is plotted as a straight line.
However, the actual load curve will not be a simple straight line but a curve.

(2) Load and Stress Calculations of Wave Washer

If the Wave Washer is assumed to be a continuous beam, the following formulas are given:

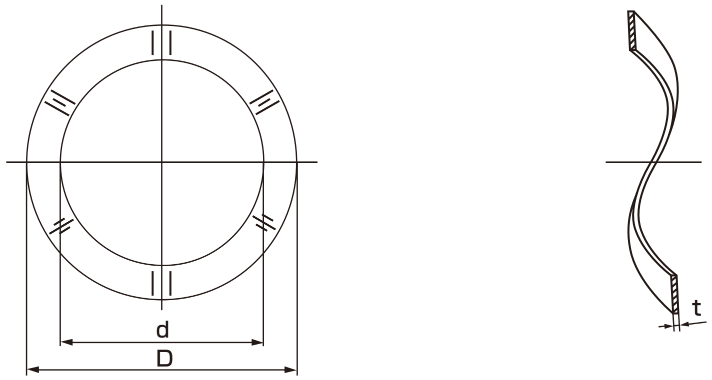


Fig. 3 Wave washer

Load

$$P = \frac{16Ebt^3N^4\delta}{\pi^3 D_m^3} \quad (3)$$

Stress

$$S = \frac{0.75\pi P D_m}{bt^2N^2} \quad (4)$$

- P : Load (N)
- S : Stress (N/mm²)
- D : Diameter of outer periphery (mm)
- d : Diameter of inner periphery (mm)
- D_m : Average diameter (mm)〔=(D+d)/2〕
- b : Rim width (mm)〔=(D−d)/2〕
- t : Plate thickness (mm)
- N : Number of waves
- δ : Amount of deflection (mm)
- E : Longitudinal elastic modulus (N/mm²) Table 1
- π : Circumference ratio

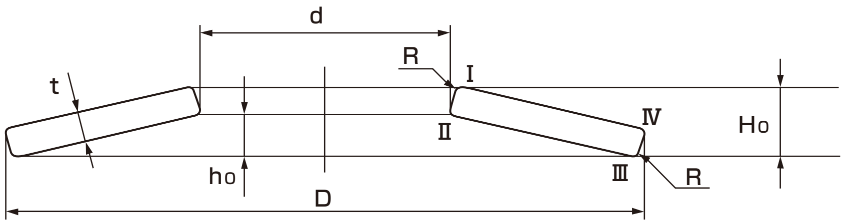
Reference for design

- To change the load by a large amount:
  Adjust the plate thickness and the number of waves.
  The load is proportional to the second power of the plate thickness and to the fourth power of the number of waves.
  (However, settling will arise if the number of waves is increased. Therefore it is better not to adjust the number much.)
- To change the load by a small amount:
  Adjust the diameters of inner and outer peripheries (rim width). The load is proportional to the rim width.

Notes

There are differences between the calculated and measured values for the formula of deflection and load. Substitution of conditions such as diameters of outer and inner peripheries gives a first-order equation of deflection and load which is plotted as a straight line.
However, the actual load curve will not be a simple straight line but a curve.

(3) Load and Stress Calculations of Dish Washer (Reference data: JIS B 2006)



- D : Diameter of outer periphery (mm)
- d : Diameter of inner periphery (mm)
- t : Plate thickness (mm)
- H_0 : Free hight (mm)
- h_0 : Total amount of deflection (H_0-t) (mm)
- E : Longitudinal elastic modulus (N/mm²)
- ν : Poisson' s ratio of material (0.3)
- P : Load(N)
- δ : Amount of deflection (mm)
- k : Load rate (N/mm)
- R : Chamfer radius of corner (mm)
- σ_{I} : Stress on position I (N/mm²)
- σ_{II} : Stress on position II (N/mm²)
- σ_{III} : Stress on position III (N/mm²)
- σ_{IV} : Stress on position IV (N/mm²)

The coefficients used for calculation are as follows:

$$\alpha = \frac{D}{d}$$

$$C_1 = \frac{1}{\pi} \cdot \frac{\left(\frac{\alpha-1}{\alpha}\right)^2}{\frac{\alpha+1}{\alpha-1} - \frac{2}{\ln\alpha}}$$

$$C_2 = \frac{1}{\pi} \cdot \frac{6}{\ln\alpha} \cdot \left(\frac{\alpha-1}{\ln\alpha} - 1\right)$$

$$C_3 = \frac{3}{\pi} \cdot \frac{\alpha-1}{\ln\alpha}$$

Including the correction item $\left(\frac{D-d}{(D-d)-3R}\right)$ that allows for round chamfering of the corner

presents the load P by the following formula:

$$P = \frac{D-d}{(D-d)-3R} \cdot \frac{4E}{1-\nu^2} \cdot \frac{t^3}{C_1 D^2} \cdot \delta \cdot \left[\left(\frac{h_0}{t} - \frac{\delta}{t}\right) \cdot \left(\frac{h_0}{t} - \frac{\delta}{2t}\right) + 1\right]$$

The stresses on the positions I, II, III and IV can be calculated according to the formulas given below. A positive value indicates tensile stress while a negative value indicates compression stress.

$$\sigma_{\mathrm{I}} = \frac{4E}{1-\nu^2} \cdot \frac{t}{C_1 D^2} \cdot \delta \cdot \left[-C_2 \cdot \left(\frac{h_0}{t} - \frac{\delta}{2t}\right) - C_3\right]$$

$$\sigma_{\mathrm{II}} = \frac{4E}{1-\nu^2} \cdot \frac{t}{C_1 D^2} \cdot \delta \cdot \left[-C_2 \cdot \left(\frac{h_0}{t} - \frac{\delta}{2t}\right) + C_3\right]$$

$$\sigma_{\mathrm{III}} = \frac{4E}{1-\nu^2} \cdot \frac{t}{\alpha C_1 D^2} \cdot \delta \cdot \left[(2C_3 - C_2) \cdot \left(\frac{h_0}{t} - \frac{\delta}{2t}\right) + C_3\right]$$

$$\sigma_{\mathrm{IV}} = \frac{4E}{1-\nu^2} \cdot \frac{t}{\alpha C_1 D^2} \cdot \delta \cdot \left[(2C_3 - C_2) \cdot \left(\frac{h_0}{t} - \frac{\delta}{2t}\right) - C_3\right]$$

The load rate of the spring is non-linear and can be calculated according to the following equation.

$$k = \frac{dP}{d\delta} = \frac{D-d}{(D-d)-3R} \cdot \frac{4E}{1-\nu^2} \cdot \frac{t^3}{C_1 D^2} \cdot \left[\left(\frac{h_0}{t}\right)^2 - 3\frac{h_0}{t} \cdot \frac{\delta}{t} + \frac{3}{2}\left(\frac{\delta}{t}\right)^2 + 1\right]$$