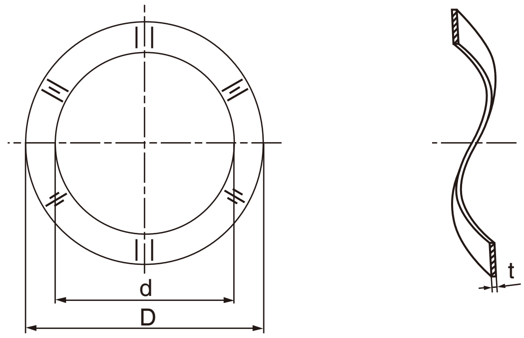
Calculations for Compressed Spring Washers (Reference)

1 Load and Stress Calculations of Wave Washer

Fig. 1 Wave Washer



Load

$$P = \frac{16Ebt^3N^4\delta}{\pi^3 D_m^3} \quad (1)$$

Stress

$$S = \frac{0.75\pi P D_m}{bt^2N^2} \quad (2)$$

Table 1 Longitudinal elastic modulus of main materials (E)

Material	Longitudinal elastic modulus (N/mm^2)
Carbon spring steel	206000
Stainless steel for spring	181000

- P: Load (N)
- S: Stress (N/mm^2)
- D: Diameter of outer periphery (mm)
- d: Diameter of inner periphery (mm)
- D_m: Average diameter (mm) [= (D + d)/2]
- b: Rim width (mm) [= (D - d)/2]
- t: Plate thickness (mm)
- N: Wave number
- δ: Amount of deflection (mm)
- E: Longitudinal elastic modulus (N/mm^2) (Table 1)
- π: Circumference ratio

Reference for design

To change the load by a large amount	Please adjust the plate thickness and wave number. The load is proportional to the cube when adjusting the plate thickness, and to the fourth power when adjusting the wave number. (However, as the number of waves increases, it becomes easier to settle, so please consider the basic three waves.)
To change the load by a small amount	Adjust the diameters of inner and outer peripheries (rim width). The load is proportional to the rim width.

Notes

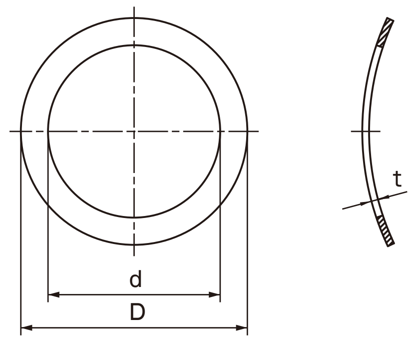
There are differences between the calculated and measured values for the formula of deflection and load. Substitution of conditions such as diameters of outer and inner peripheries gives a first-order equation of deflection and load which is plotted as a straight line. However, the actual load curve will not be a simple straight line but a curve.

Calculations for Compressed Spring Washers (Reference)

OCHIAI OF INDUSTRIAL FASTENERS

2 Load and Stress Calculations of Curved Washer

Fig. 1 Curved Washer



Load

$$P = \frac{4K_1Et^3\delta}{D^2} \quad (1)$$

Stress

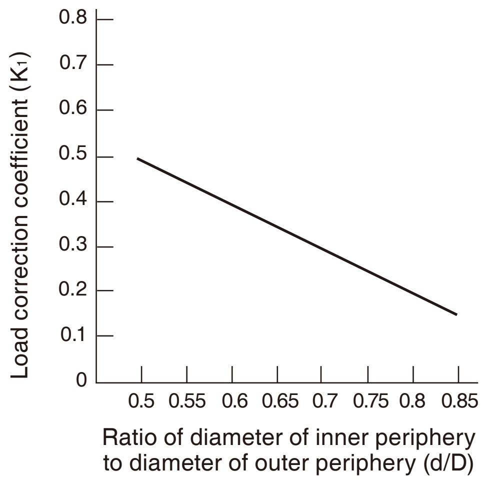
$$S = \frac{1.5P}{K_1t^2} \quad (2)$$

- P: Load (N)
- S: Stress (N/mm²)
- D: Diameter of outer periphery (mm)
- d: Diameter of inner periphery (mm)
- t: Plate thickness (mm)
- δ: Amount of deflection (mm)
- E: Longitudinal elastic modulus (N/mm²) (Table 1)
- K_1: Load correction coefficient [= 1 - d/D] (Table 2)

Table 1 Longitudinal elastic modulus of main materials (E)

Material	Longitudinal elastic modulus (N/mm²)
Carbon spring steel	206000
Stainless steel for spring	181000

Table 2



Notes

There are differences between the calculated and measured values for the formula of deflection and load. Substitution of conditions such as diameters of outer and inner peripheries gives a first-order equation of deflection and load which is plotted as a straight line. However, the actual load curve will not be a simple straight line but a curve.

3 Load and Stress Calculations of Dish Spring (Reference data: JIS B 2706)

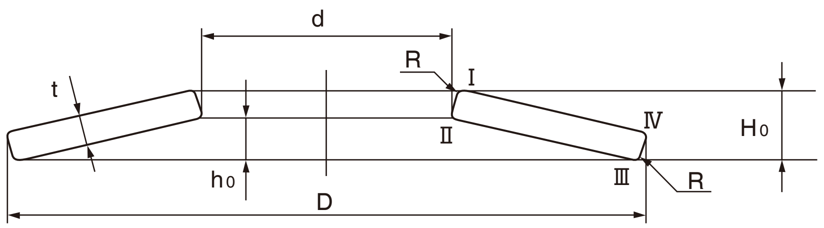


Fig. 3 Dish Spring

- D: Diameter of outer periphery (mm)
- d: Diameter of inner periphery (mm)
- t: Plate thickness (mm)
- H_0: Free height (mm)
- h_0: Total amount of deflection (H_0 - t) (mm)
- E: Longitudinal elastic modulus (N/mm²) (Table 1)
- ν: Poisson's ratio of material (0.3)
- P: Load (N)
- δ: Amount of deflection (mm)
- k: Load rate (N/mm)
- R: Chamfer radius of corner (mm)
- σI: Stress on position I (N/mm²)
- σII: Stress on position II (N/mm²)
- σIII: Stress on position III (N/mm²)
- σIV: Stress on position IV (N/mm²)

The coefficients used for calculation are as follows:

$$\alpha = \frac{D}{d}$$

$$C_1 = \frac{1}{\pi} \cdot \frac{\left(\frac{\alpha - 1}{\alpha}\right)^2}{\frac{\alpha + 1}{\alpha - 1} - \frac{2}{\ln \alpha}}$$

$$C_2 = \frac{1}{\pi} \cdot \frac{6}{\ln \alpha} \cdot \left(\frac{\alpha - 1}{\ln \alpha} - 1\right)$$

$$C_3 = \frac{3}{\pi} \cdot \frac{\alpha - 1}{\ln \alpha}$$

Including the correction item $\left(\frac{D - d}{(D - d) - 3R}\right)$ that allows for round chamfering of the corner presents the load P by the following formula:

$$P = \frac{D - d}{(D - d) - 3R} \cdot \frac{4E}{1 - \nu^2} \cdot \frac{t^3}{C_1D^2} \cdot \delta \cdot \left[\left(\frac{h_0}{t} - \frac{\delta}{t}\right) \cdot \left(\frac{h_0}{t} - \frac{\delta}{2t}\right) + 1\right]$$

The stresses on the positions I, II, III and IV can be calculated according to the formulas given below. A positive value indicates tensile stress while a negative value indicates compression stress.

$$\sigma_{I} = \frac{4E}{1 - \nu^2} \cdot \frac{t}{C_1D^2} \cdot \delta \cdot \left[- C_2 \cdot \left(\frac{h_0}{t} - \frac{\delta}{2t}\right) - C_3\right]$$

$$\sigma_{II} = \frac{4E}{1 - \nu^2} \cdot \frac{t}{C_1D^2} \cdot \delta \cdot \left[- C_2 \cdot \left(\frac{h_0}{t} - \frac{\delta}{2t}\right) - C_3\right]$$

$$\sigma_{III} = \frac{4E}{1 - \nu^2} \cdot \frac{t}{\alpha C_1D^2} \cdot \delta \cdot \left[(2C_3 - C_2) \cdot \left(\frac{h_0}{t} - \frac{\delta}{2t}\right) + C_3\right]$$

$$\sigma_{IV} = \frac{4E}{1 - \nu^2} \cdot \frac{t}{\alpha C_1D^2} \cdot \delta \cdot \left[(2C_3 - C_2) \cdot \left(\frac{h_0}{t} - \frac{\delta}{2t}\right) - C_3\right]$$

The load rate of the spring is non-linear and can be calculated according to the following equation.

$$k = \frac{dP}{d\delta} = \frac{D - d}{(D - d) - 3R} \cdot \frac{4E}{1 - \nu^2} \cdot \frac{t^3}{C_1D^2} \cdot \left[\left(\frac{h_0}{t}\right)^2 - 3\frac{h_0}{t} \cdot \frac{\delta}{t} + \frac{3}{2}\left(\frac{\delta}{t}\right)^2 + 1\right]$$